The New World of Neutrino Physics

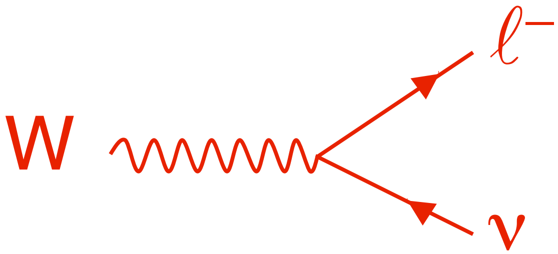
Part One

Boris Kayser
Fermilab
Feb. 14, 2006

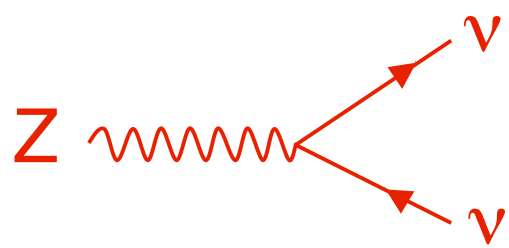
How Can the Standard Model be Modified to Include Neutrino Masses?

Majorana Neutrinos or Dirac Neutrinos?

The S(tandard) M(odel)



and

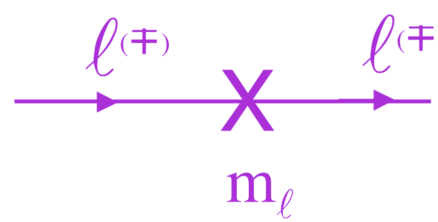


couplings conserve the Lepton Number L
defined by—

$$L(\nu) = L(\ell^-) = -\,L(\bar{\nu}) = -\,L\,(\ell^+) = 1.$$

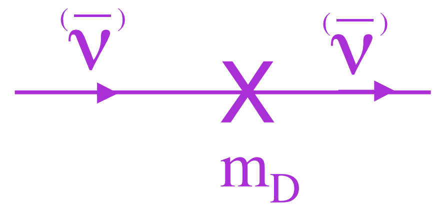
So do the Dirac charged-lepton mass terms

$$m_\ell \overline{\ell_L} \ell_R$$



- Original SM: $m_\nu = 0$.
- Why not add a Dirac mass term,

$$m_D \overline{\nu}_L \nu_R$$



Then everything conserves L, so for each mass eigenstate ν_i,

$\overline{\nu}_i \neq \nu_i$ (Dirac neutrinos)

$[L(\overline{\nu}_i) = -L(\nu_i)]$

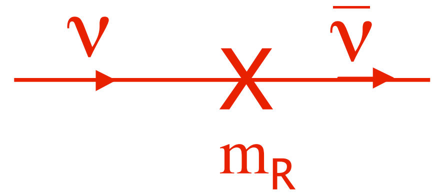
- The SM contains no ν_R field, only ν_L.

To add the Dirac mass term, we had to add ν_R to the SM.

Unlike ν_L, ν_R carries no Electroweak Isospin.

Thus, no SM principle prevents the occurrence of the Majorana mass term

$$m_R \overline{\nu_R}^c \nu_R$$



But this does not conserve L, so now

$$\bar{\nu}_i = \nu_i \qquad \text{(Majorana neutrinos)}$$

[No conserved L to distinguish $\bar{\nu}_i$ from ν_i]

We note that $\bar{\nu}_i = \nu_i$ means —

$$\bar{\nu}_i(h) = \nu_i(h)$$

helicity

Many Theorists Expect Majorana Masses

The Standard Model (SM) is defined by the fields it contains, its symmetries (notably Electroweak Isospin Invariance), and its renormalizability.

Leaving neutrino masses aside, anything allowed by the SM symmetries occurs in nature.

If this is also true for neutrino masses, then neutrinos have Majorana masses.

- The presence of Majorana masses
- $\overline{\nu_i} = \nu_i$ (Majorana neutrinos)
- L not conserved

— are all equivalent

Any one implies the other two.

The See-Saw Mechanism

For a *Dirac* neutrino mass eigenstate ν of mass m, the mass term in the Lagrangian density is —

$$L_m = -m\bar{\nu}\nu$$

Then —

$$\langle \nu \text{ at rest} \,|H_m|\, \nu \text{ at rest}\rangle = \langle \nu \text{ at rest} \left| m\int d^3x\; \bar{\nu}\nu \right| \nu \text{ at rest} \rangle = m$$

Hamiltonian

For a *Majorana* neutrino mass eigenstate ν of mass m, the mass term in the Lagrangian density is —

$$L_m = -\frac{m}{2}\bar{\nu}\nu$$

with $\underbrace{\nu^c = \text{(phase factor) x } \nu}_{\text{Antineutrino = Neutrino}}$

Then —

$$\langle \nu \text{ at rest} | H_m | \nu \text{ at rest} \rangle = \langle \nu \text{ at rest} \left| \frac{m}{2} \int d^3x\, \bar{\nu}\nu \right| \nu \text{ at rest} \rangle = m$$

{The matrix element of $\bar{\nu}\nu$ is doubled in the Majorana case.}

Chiral fields:

Chirally left- and right-handed fermion fields satisfy the constraints —

$$P_L f_L \equiv \frac{(1-\gamma_5)}{2} f_L = f_L \qquad \text{and} \qquad P_R f_R \equiv \frac{(1+\gamma_5)}{2} f_R = f_R$$

For a *massless* fermion, chirality = helicity.

In the Standard Model (SM), only chirally left-handed fermion fields couple to the W boson.

Therefore, it is convenient to express the SM in terms of "underlying" chiral fields.

Expressed in terms of chiral fields, any mass term connects only fields of *opposite* chirality:

$$\bar{g}_R f_L$$

Chiral fermion fields

$$\bar{j}_L k_L = \bar{j}_R k_R = 0$$

Chiral fermion fields

For example —

$$\bar{j}_L k_L = \overline{\left(\frac{1-\gamma_5}{2}\right) j}\left(\frac{1-\gamma_5}{2}\right) k = \bar{j}\left(\frac{1+\gamma_5}{2}\right)\left(\frac{1-\gamma_5}{2}\right) k = 0$$

Note: Charge conjugating a chiral field reverses its chirality.

Dirac Mass Term

For quarks, charged leptons and *maybe* neutrinos.

Suppose $\nu_L{}^0$ and $\nu_R{}^0$ are underlying chiral fields in terms of which the SM, extended to include neutrino mass, is written.

The Dirac mass term is then —

$$L_D = -m_D \overline{\nu_R{}^0} \nu_L{}^0 + \text{ h.c. } = -m_D (\overline{\nu_R{}^0} \nu_L{}^0 + \overline{\nu_L{}^0} \nu_R{}^0)$$

In terms of $\nu \equiv \nu_L{}^0 + \nu_R{}^0$, $L_D = -m_D \bar{\nu}\nu$, since

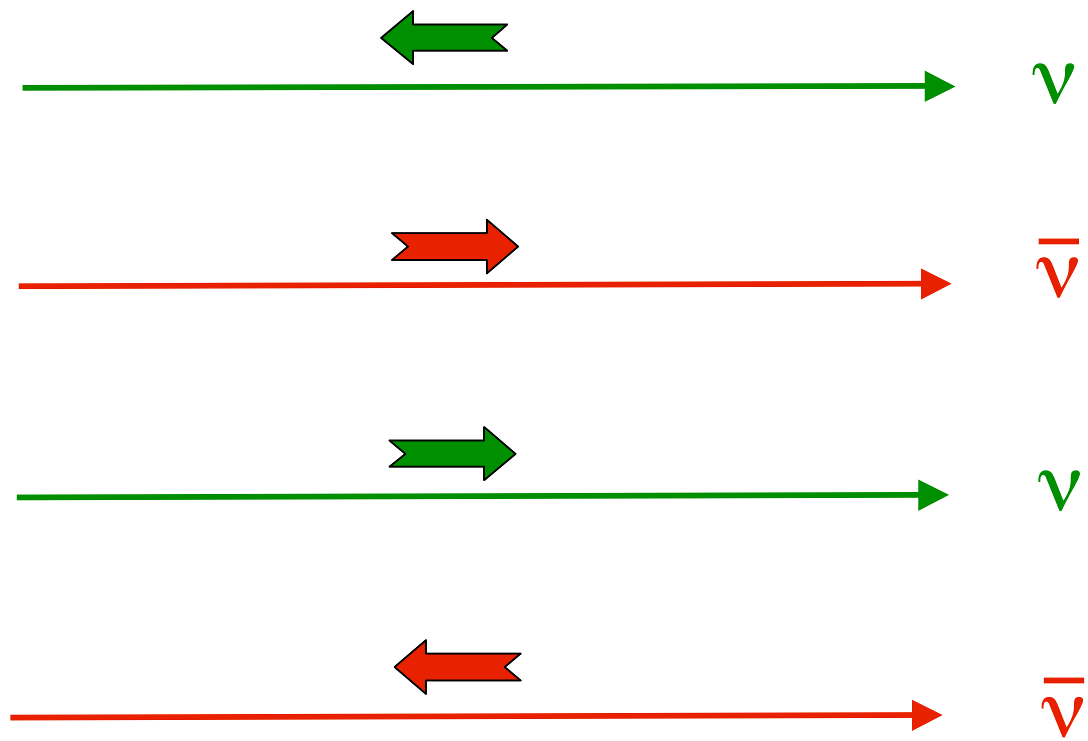
$$\bar{\nu}\nu = \overline{\left(\nu_L{}^0 + \nu_R{}^0\right)}\left(\nu_L{}^0 + \nu_R{}^0\right) = \overline{\nu_R{}^0}\nu_L{}^0 + \overline{\nu_L{}^0}\nu_R{}^0$$

ν is the mass eigenstate, and has mass m_D.

We have 4 mass-degenerate states:

L is conserved.

$\bar{\nu} \neq \nu$



This collection of 4 states is a Dirac neutrino plus its antineutrino.

Majorana Mass Term

For neutrinos only.

Suppose $\nu_R{}^0$ is an electroweak singlet chiral field.

The right-handed Majorana mass term is then —

$$L_R = -\frac{m_R}{2}\overline{\left(\nu_R{}^0\right)^c}\nu_R{}^0 + \text{ h.c. } = -\frac{m_R}{2}\left[\overline{\left(\nu_R{}^0\right)^c}\nu_R{}^0 + \overline{\nu_R{}^0}\left(\nu_R{}^0\right)^c\right]$$

In terms of $\nu \equiv \nu_R{}^0 + \left(\nu_R{}^0\right)^c$, $L_R = -\frac{m_R}{2}\bar{\nu}\nu$, since

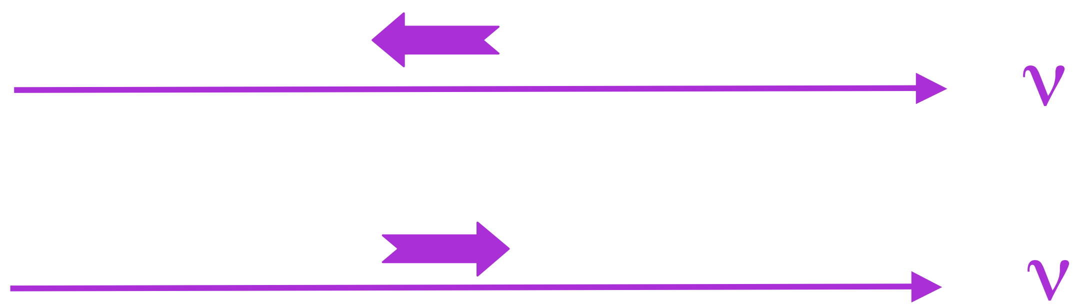
$$\bar{\nu}\nu = \overline{\left[\nu_R{}^0 + \left(\nu_R{}^0\right)^c\right]}\left[\nu_R{}^0 + \left(\nu_R{}^0\right)^c\right] = \overline{\left(\nu_R{}^0\right)^c}\nu_R{}^0 + \overline{\nu_R{}^0}\left(\nu_R{}^0\right)^c$$

ν is the mass eigenstate, and has mass m_R.

$$\nu^c = \left[\nu_R{}^0 + \left(\nu_R{}^0\right)^c\right]^c = \left(\nu_R{}^0\right)^c + \nu_R{}^0 = \nu$$

Thus, ν is its own antiparticle. It is a Majorana neutrino.

We have only 2 mass-degenerate states:



The See-Saw

We include *both* Majorana and Dirac mass terms:

$$L_m = -m_D \overline{\nu_R{}^0}\nu_L{}^0 - \frac{m_R}{2}\overline{\left(\nu_R{}^0\right)^c}\nu_R{}^0 + \text{ h.c.}$$

$$= -\frac{1}{2}\left[\overline{\left(\nu_L{}^0\right)^c}, \overline{\nu_R{}^0}\right]\begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix}\begin{bmatrix} \nu_L{}^0 \\ \left(\nu_R{}^0\right)^c \end{bmatrix} + \text{ h.c.}$$

We have used $\overline{\left(\nu_L{}^0\right)^c} m_D \left(\nu_R{}^0\right)^c = \overline{\nu_R{}^0} m_D \nu_L{}^0$.

$M_\nu = \begin{bmatrix} 0 & m_D \\ m_D & m_R \end{bmatrix}$ is called the neutrino mass matrix.

No SM principle prevents m_R from being extremely large.

But we expect m_D to be of the same order as the masses of the quarks and charged leptons.

Thus, we assume that $m_R >> m_D$.

M_ν can be diagonalized by the transformation —

$$Z^T M_\nu Z = D_\nu$$

With $\rho \equiv m_D / m_R << 1$,

Makes eigenvalues positive

$$Z \cong \begin{bmatrix} 1 & \rho \\ -\rho & 1 \end{bmatrix} \begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$D_\nu \cong \begin{bmatrix} m_D{}^2 / m_R & 0 \\ 0 & m_R \end{bmatrix}$$

Define $\begin{bmatrix} \nu_L \\ N_L \end{bmatrix} \equiv Z^{-1} \begin{bmatrix} {\nu_L}^0 \\ \left({\nu_R}^0\right)^c \end{bmatrix}$ and $\begin{bmatrix} \nu \\ N \end{bmatrix} \equiv \begin{bmatrix} \nu_L + \left(\nu_L\right)^c \\ N_L + \left(N_L\right)^c \end{bmatrix}$.

Majorana neutrinos

Then —

$$L_m = -\frac{1}{2}\frac{{m_D}^2}{m_R}\overline{\nu}\nu - \frac{1}{2}m_R\overline{N}N$$

Mass of ν

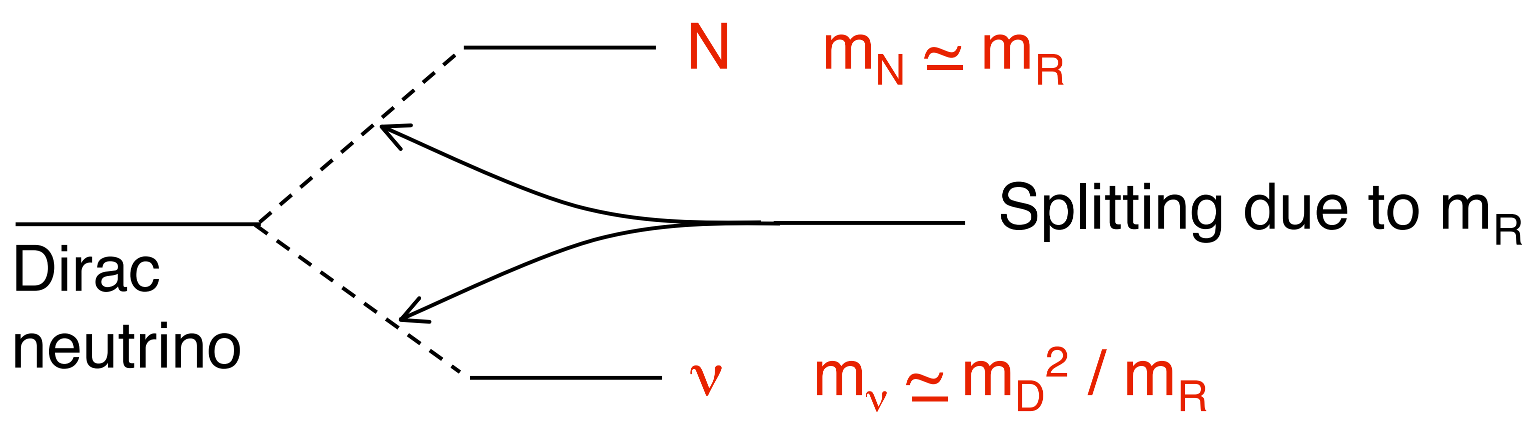
Mass of N

(Mass of ν) x (Mass of N) = ${m_D}^2 \sim {m_{\text{quark or lepton}}}^2$

The See-Saw Relation

What Happened?

The Majorana mass term split a Dirac neutrino into two Majorana neutrinos.



Predictions of the See-Saw

- Each $\bar{\nu}_i = \nu_i$ (Majorana neutrinos)
- The light neutrinos have heavy partners N

How heavy??

$$m_N \sim \frac{m^2{}_{top}}{m_\nu} \sim \frac{m^2{}_{top}}{0.05\text{ eV}} \sim 10^{15}\text{ GeV}$$

Near the GUT scale.


The Open Questions

Neutrinos and the New Paradigm

- What are the masses of the neutrinos?

  Is the spectrum like  or  ?

- What is the pattern of mixing among the different types of neutrinos?

  What is θ_{13}? Is θ_{23} maximal?

- Are neutrinos their own antiparticles?

- Do neutrinos violate the symmetry CP? Is
$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$?

Neutrinos and the Unexpected

- Are there "sterile" neutrinos?

- Do neutrinos have unexpected or exotic properties?

We must be alert to further surprises.

- What can neutrinos tell us about the models of new physics beyond the Standard Model?

The See-Saw Mechanism relates ν masses to physics at the high-mass scale where the forces become unified.

A signature feature of the See-Saw is that $\bar{\nu} = \nu$.

Neutrinos and the Cosmos

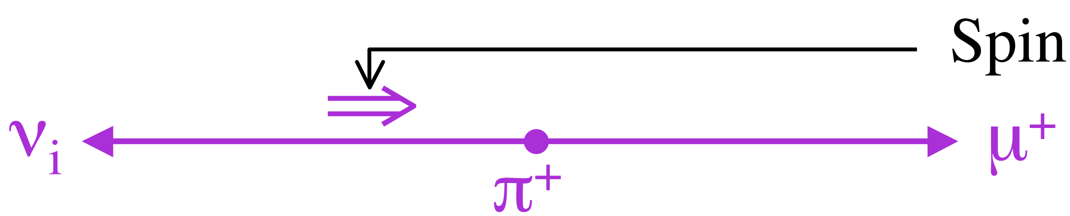
- What is the role of neutrinos in shaping the universe?
- Is CP violation by neutrinos the key to understanding the matter – antimatter asymmetry of the universe?
- What can neutrinos reveal about the deep interior of the earth and sun, and about supernovae and other ultra high energy astrophysical phenomena?

How Can We Demonstrate That $\overline{\nu}_i = \nu_i$?

We assume neutrino interactions are correctly described by the SM. Then the interactions conserve L ($\nu \rightarrow \ell^-$; $\bar{\nu} \rightarrow \ell^+$).

An Idea that Does Not Work
[and illustrates why most ideas do not work]

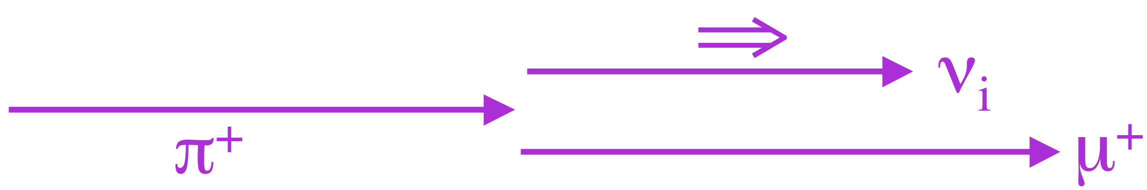
Produce a ν_i via—



Pion Rest Frame

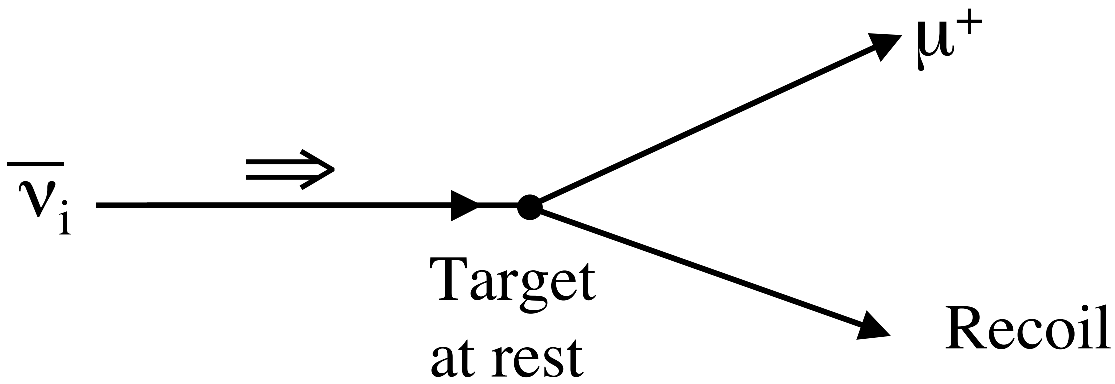
Give the neutrino a Boost:

$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$



Lab. Frame

The SM weak interaction causes—



$\nu_i = \overline{\nu}_i$ means that $\nu_i(h) = \overline{\nu}_i(h)$.

helicity

If $\nu_i \xrightarrow{\Rightarrow} = \overline{\nu}_i \xrightarrow{\Rightarrow}$,

our $\nu_i \xrightarrow{\Rightarrow}$ will make μ^+ too.

Minor Technical Difficulties

$$\beta_\pi(\text{Lab}) > \beta_\nu(\pi \text{ Rest Frame})$$

$$\Rightarrow \frac{E_\pi(\text{Lab})}{m_\pi} > \frac{E_\nu(\pi \text{ Rest Frame})}{m_{\nu_i}}$$

$$\Rightarrow E_\pi(\text{Lab}) \gtrsim 10^5 \text{ TeV if } m_{\nu_i} \sim 0.05 \text{ eV}$$

Fraction of all π – decay ν_i that get helicity flipped

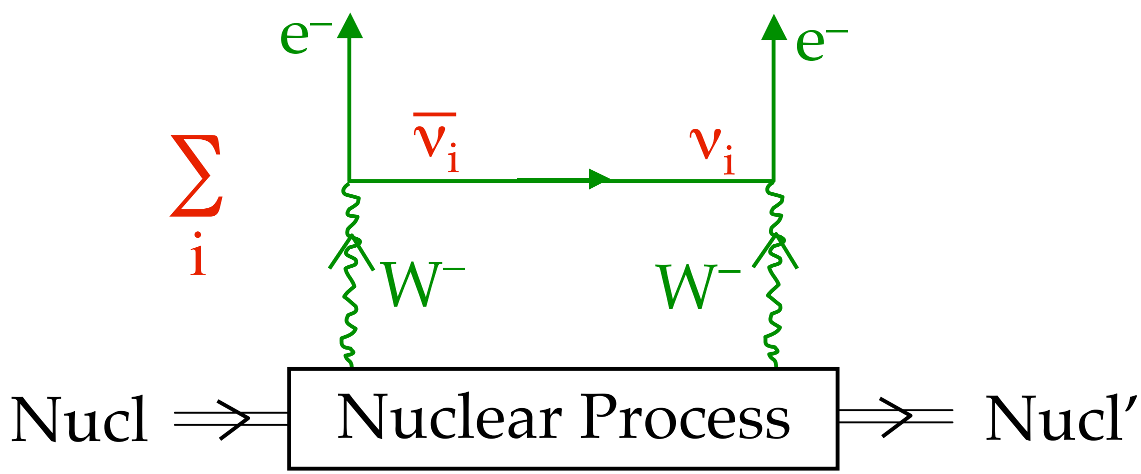
$$\approx \left(\frac{m_{\nu_i}}{E_\nu(\pi \text{ Rest Frame})} \right)^2 \sim 10^{-18} \text{ if } m_{\nu_i} \sim 0.05 \text{ eV}$$

Since L-violation comes only from Majorana neutrino *masses*, any attempt to observe it will be at the mercy of the neutrino masses.

(BK & Stodolsky)

The Idea That Can Work —

Neutrinoless Double Beta Decay [$0\nu\beta\beta$]

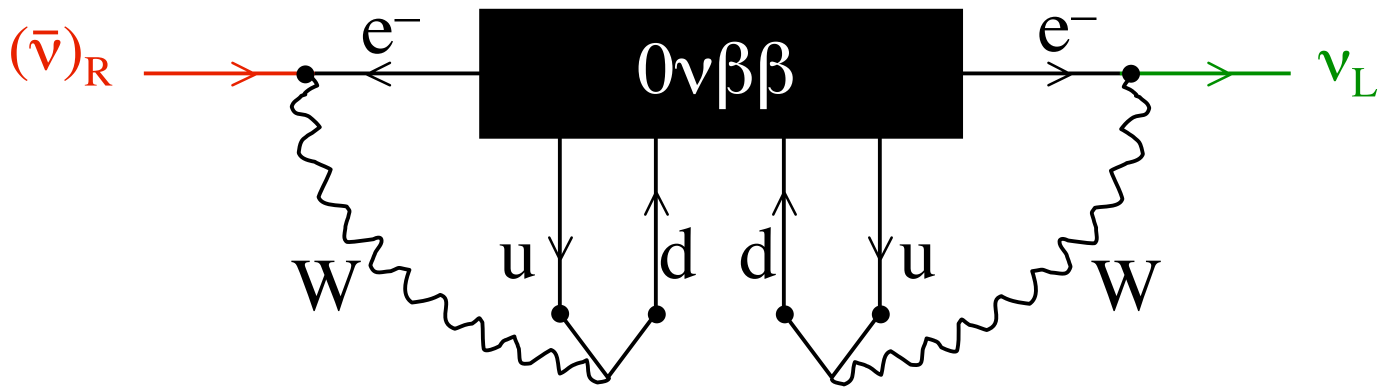


By avoiding competition, this process can cope with the small neutrino masses.

Observation would imply $\not{L}$ and $\bar{\nu}_i = \nu_i$.

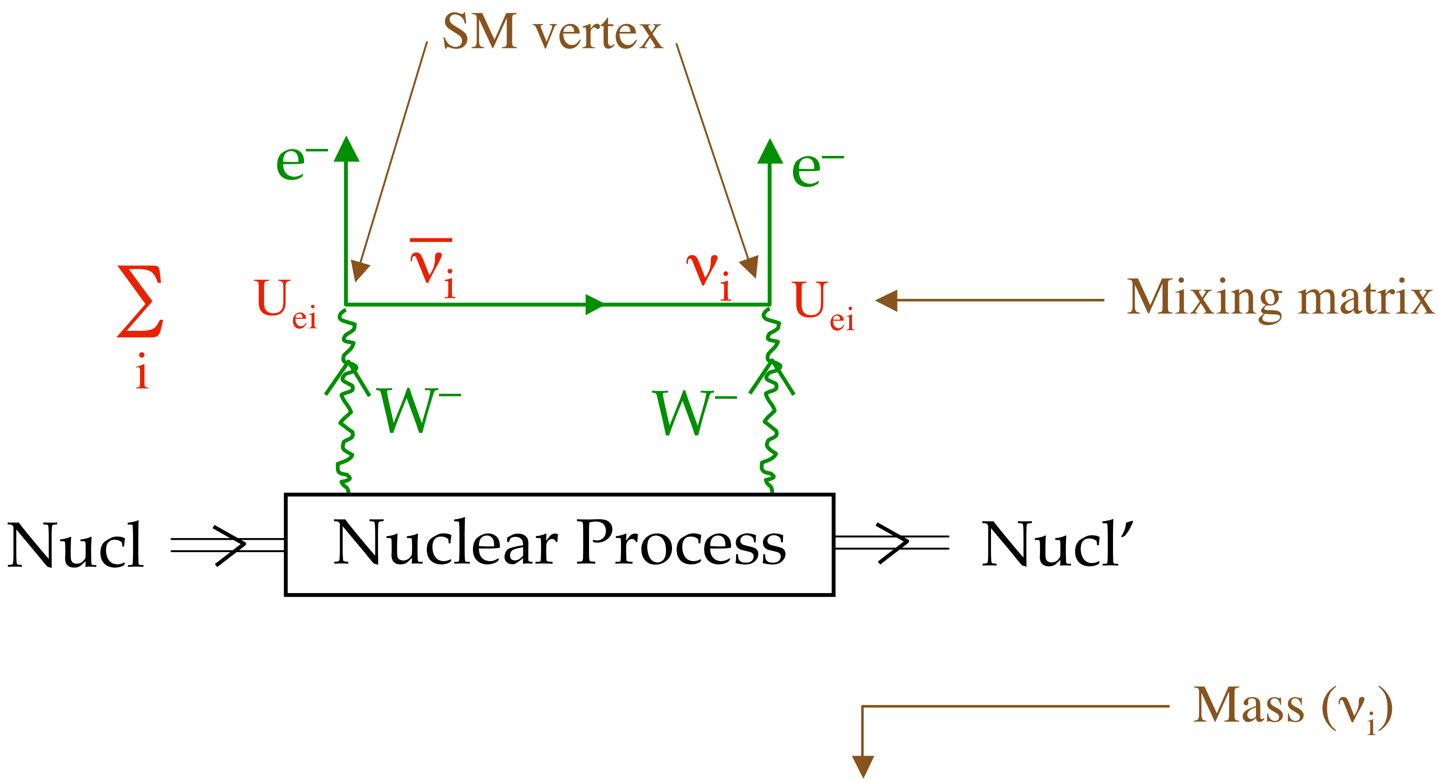
Whatever diagrams cause $0\nu\beta\beta$, its observation would imply the existence of a Majorana mass term:

Schechter and Valle



$(\bar{\nu})_R \rightarrow \nu_L$: A Majorana mass term

In —



the $\bar{\nu}_i$ is emitted [RH + O{m_i/E}LH].

Thus, Amp [ν_i contribution] $\propto m_i$

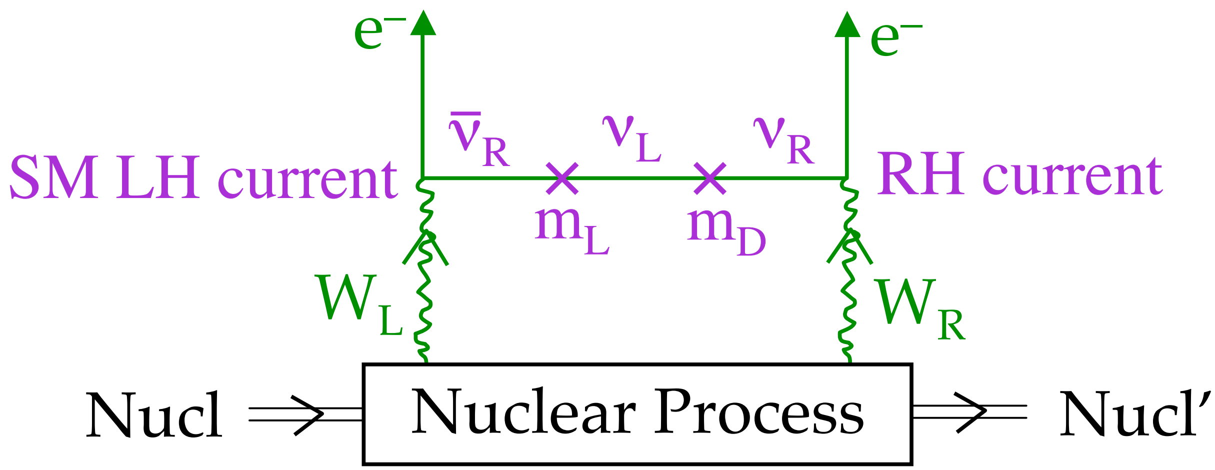
$$\mathrm{Amp}[0\nu\beta\beta] \propto \left|\sum_i m_i U_{ei}^2\right| \equiv m_{\beta\beta}$$

The proportionality of $0\nu\beta\beta$ to mass is no surprise.

$0\nu\beta\beta$ violates L. But the SM interactions conserve L.

The L – violation in $0\nu\beta\beta$ comes from underlying **Majorana** mass terms.

Wouldn't the dependence on neutrino mass be eliminated by a Right-Handed Current?



The SM LH current does not violate L.

An identical current, but of opposite handedness, wouldn't violate L either.

We still need the L-violating Majorana neutrino mass to make this process occur.

With a RH current at one vertex,

$$\text{Amp}[0\nu\beta\beta] \propto (\nu \text{ mass})^2 .$$

Contributions with a RH current at one vertex
are not likely to be significant.

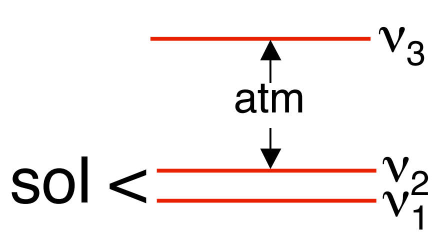
$$\left\{ \begin{array}{c} \text{BK, Petcov, Rosen} \\ \text{Enqvist, Maalampi, Mursula} \end{array} \right\}$$

How Large is $m_{\beta\beta}$?

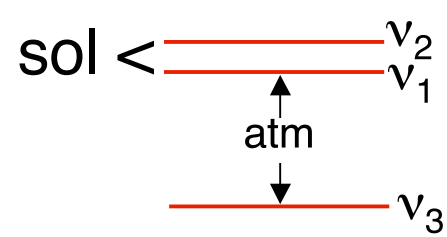
How sensitive need an experiment be?

Suppose there are only 3 neutrino mass eigenstates. (More might help.)

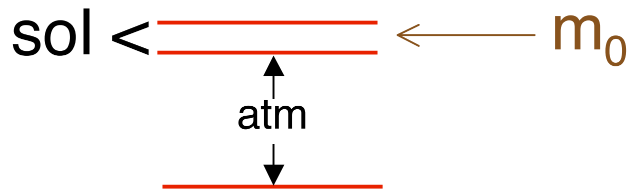
Then the spectrum looks like —



or



If the spectrum looks like —



then–

Majorana ~~CP~~ phases

$$m_{\beta\beta} \cong m_0\left[1 - \sin^2 2\theta_\odot \sin^2\left(\frac{\alpha_2 - \alpha_1}{2}\right)\right]^{1/2} \quad .$$

Solar mixing angle

$$m_0 \cos 2\theta_\odot \leq m_{\beta\beta} \leq m_0$$

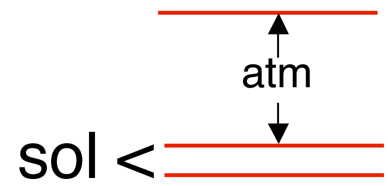
At 90% CL,

$m_0 > 40$ meV (SuperK); $\cos 2\theta_\odot > 0.28$ (SNO),

so

$$m_{\beta\beta} > 11 \text{ meV} \ .$$

If the spectrum looks like $\longrightarrow$



then —

$$0 < m_{\beta\beta} < \text{Present Bound } [(0.3\text{–}1.0)\text{ eV}]\ .$$

(Petcov *et al.*)

Analyses of $m_{\beta\beta}$ vs. Neutrino Parameters

Barger, Bilenky, Farzan, Giunti, Glashow, Grimus, BK, Kim, Klapdor-Kleingrothaus, Langacker, Marfatia, Monteno, Murayama, Pascoli, Päs, Peña-Garay, Peres, Petcov, Rodejohann, Smirnov, Vissani, Whisnant, Wolfenstein,

Review of $\beta\beta$ Decay: Elliott & Vogel

Evidence for $0\nu\beta\beta$ with $m_{\beta\beta} = (0.05 - 0.84)$ eV?

Klapdor-Kleingrothaus